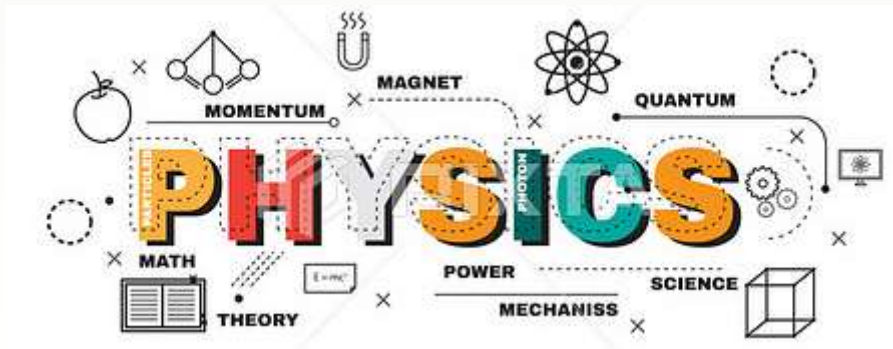




CHAPTER [3]

Motion in Two or Three Dimensions

[PHYS 101] · General Physics I

Kuwait University

PREPARED BY

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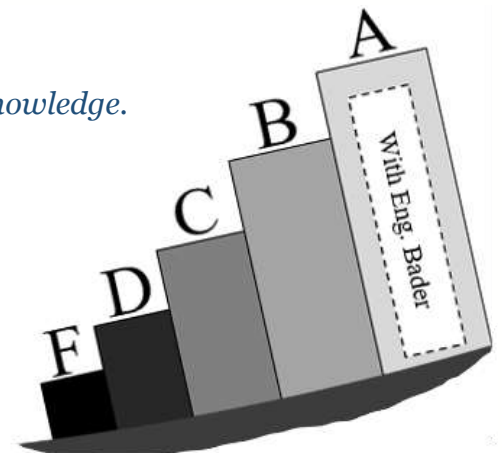
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Imagination is more important than knowledge.

— Albert Einstein



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PHYS101 – Chapter 3 Summary

Motion in Two or Three Dimensions

1) Position, Displacement, and Velocity Vectors

Position vector: $\vec{r} = x \hat{i} + y \hat{j}$ ($\vec{r}_i = x_i \hat{i} + y_i \hat{j}$, $\vec{r}_f = x_f \hat{i} + y_f \hat{j}$)

Displacement vector: $\Delta \vec{r} = \vec{r}_f - \vec{r}_i = (x_f - x_i) \hat{i} + (y_f - y_i) \hat{j} = \Delta x \hat{i} + \Delta y \hat{j}$

Average velocity: $\vec{v}_{av} = \Delta \vec{r} / t = (\Delta x / t) \hat{i} + (\Delta y / t) \hat{j} = v_{av-x} \hat{i} + v_{av-y} \hat{j}$

Instantaneous velocity: $\vec{v} = d\vec{r}/dt = (dx/dt) \hat{i} + (dy/dt) \hat{j} = v_x \hat{i} + v_y \hat{j}$

Instantaneous speed: $v = |\vec{v}| = \sqrt{(v_x^2 + v_y^2)}$

2) The Acceleration Vector

Average acceleration: $\vec{a}_{av} = \Delta \vec{v} / t = (\Delta v_x / t) \hat{i} + (\Delta v_y / t) \hat{j} = a_{av-x} \hat{i} + a_{av-y} \hat{j}$

Instantaneous acceleration: $\vec{a} = d\vec{v}/dt = (dv_x/dt) \hat{i} + (dv_y/dt) \hat{j} = a_x \hat{i} + a_y \hat{j}$

3) Parallel & Perpendicular Components of Acceleration

- Parallel component $a_{\parallel}$ — describes the change in the particle's speed (tangent to the path).
- Perpendicular component $a_{\perp}$ — describes the change in the direction of motion (normal to the path).

4) Motion in 2D with Constant Acceleration & Projectile Motion

If $\vec{a} = a_x \hat{i} + a_y \hat{j} = \text{constant}$, the x- and y-motions are independent:

x-motion	y-motion
$v_{xf} = v_{xi} + a_x t$	$v_{yf} = v_{yi} + a_y t$
$x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$ ($\Delta x = v_{xi} t + \frac{1}{2} a_x t^2$)	$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$ ($\Delta y = v_{yi} t + \frac{1}{2} a_y t^2$)
$v_{xf}^2 = v_{xi}^2 + 2a_x \Delta x$	$v_{yf}^2 = v_{yi}^2 + 2a_y \Delta y$
$\Delta x = \frac{1}{2} (v_{xi} + v_{xf}) t$	$\Delta y = \frac{1}{2} (v_{yi} + v_{yf}) t$

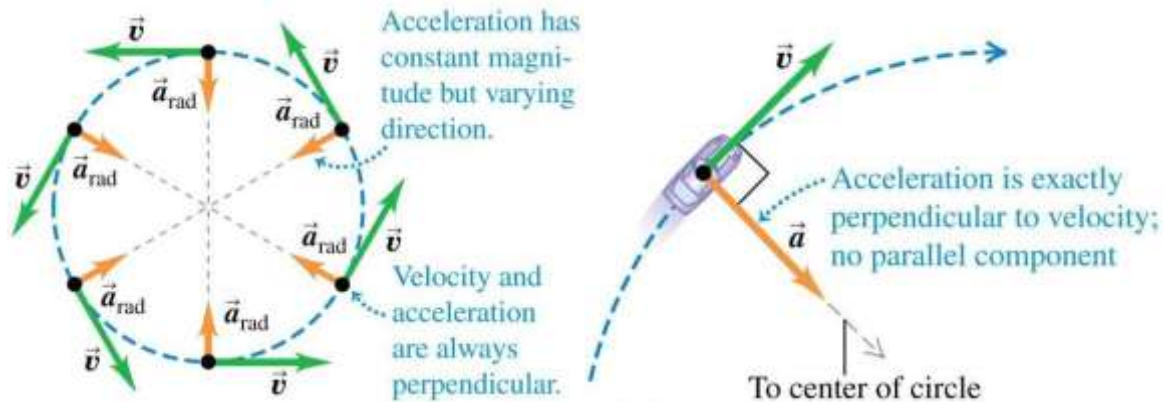
Projectile motion

(ignore air resistance): $a_x = 0$, $a_y = -g = -10 \text{ m/s}^2$ (or -9.8 m/s^2).

$v_{xi} = v_i \cos \theta$, $v_{yi} = v_i \sin \theta$ (θ measured above the horizontal at launch)

x-motion (constant velocity)	y-motion (free fall)
$v_{xf} = v_{xi}$ (unchanged)	$v_{yf} = v_{yi} - g t$
$x_f = x_i + v_{xi} t$ ($\Delta x = v_{xi} t$)	$y_f = y_i + v_{yi} t - \frac{1}{2} g t^2$ ($\Delta y = v_{yi} t - \frac{1}{2} g t^2$)
—	$v_{yf}^2 = v_{yi}^2 - 2g \Delta y$
—	$\Delta y = \frac{1}{2} (v_{yi} + v_{yf}) t$

5) Motion in a Circle – Uniform Circular Motion



Uniform circular motion: motion with constant speed along a circular path, $|\vec{v}| = \text{constant}$.

The instantaneous acceleration always points toward the center of the circle — the centripetal acceleration:

$$a_{\text{rad}} = v^2 / R$$

Period T (time for one revolution): $v = 2\pi R / T$ and $a_{\text{rad}} = 4\pi^2 R / T^2$

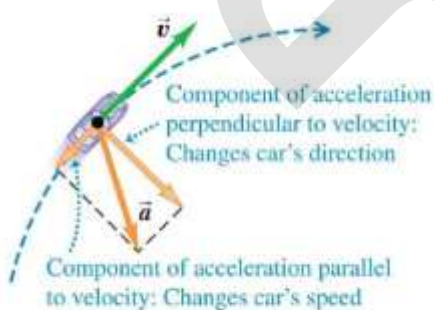
6) Motion in a Circle – Non-Uniform Circular Motion

Non-uniform circular motion: motion with varying speed along a circular path, $|\vec{v}| \neq \text{constant}$.

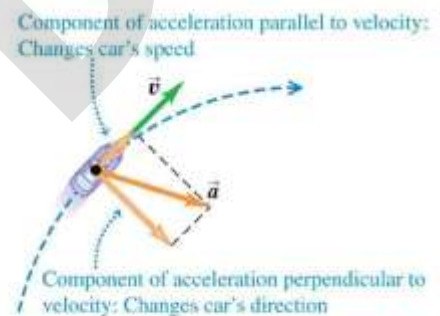
Radial component (still present): $a_{\text{rad}} = v^2 / R$ — changes the direction of motion.

Tangential component a_{tan} — parallel/antiparallel to the instantaneous velocity; changes the particle's speed.

- If $\vec{a}$ has a component parallel to $\vec{v}$ (same direction) $\rightarrow$ the object speeds up along the circle.
- If $\vec{a}$ has a component antiparallel to $\vec{v}$ (opposite direction) $\rightarrow$ the object slows down along the circle.



• Car slowing down along a circular path



• Car speeding up along a circular path

Slides Questions Chapter 3

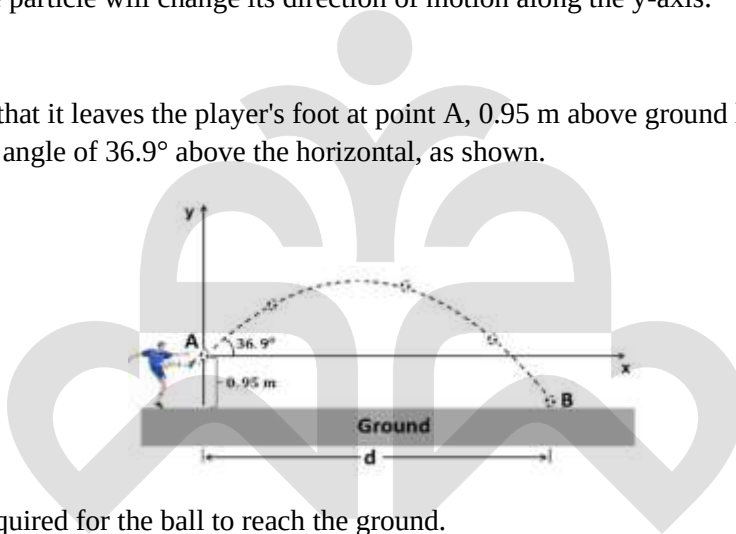
Q1: A particle is moving in the x-y plane, its position vector is given by $\vec{r} = [(6 + 3t)\hat{i} + (5 + 2t - 2t^2)\hat{j}]$ m, where t is measured in seconds.

- (a) Find its position vector and velocity vector at $t = 0$ s (in unit vector notation).
- (b) Find the acceleration of the particle (in unit vector notation).
- (c) How long (in s) will it take the particle to reach the maximum y-coordinate?

Q2: A particle moves in the x-y plane, starting from the origin with a velocity of $\vec{v} = (3\hat{i} + 12\hat{j})$ m/s and a constant acceleration of $\vec{a} = (1\hat{i} - 2\hat{j})$ m/s². Find:

- (a) The position of the particle at $t = 3$ s.
- (b) The velocity and speed of the particle at $t = 2$ s.
- (c) At what time the particle will change its direction of motion along the y-axis.

Q3: A ball is shot such that it leaves the player's foot at point A, 0.95 m above ground level. The initial speed of the ball is 15 m/s at an angle of 36.9° above the horizontal, as shown.

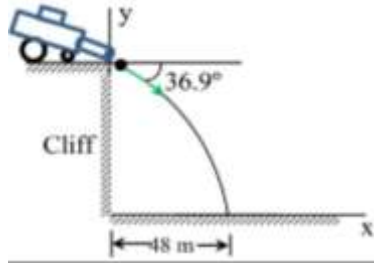


- (a) Find the time required for the ball to reach the ground.
- (b) Find the horizontal distance (d) between point A and point B.
- (c) Find the velocity of the ball just before it strikes the ground (at point B), in unit vector notation.

Q4: A boy kicks a ball from ground level at an angle θ above the horizontal to a wall 7.5 m away, as shown. If the initial speed of the ball is 14 m/s and it takes 2 s to reach the wall, at what height h (in m) does the ball strike the wall? Ignore air resistance.

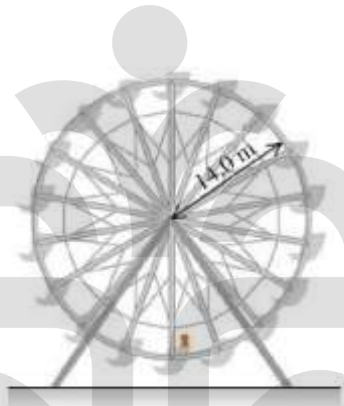


Q5: A cannon on the top of a cliff fires a ball 36.9° below the horizontal direction, as shown. The ball hits the ground after 2 s at a horizontal distance of 48 m from the cliff.



- (a) With what initial speed (in m/s) does the cannon fire the ball?
- (b) With what velocity (in m/s) does the ball hit the ground, in unit vector notation?
- (c) What is the height (in m) of the cliff?

Q6: A Ferris wheel with radius 14 m is turning about a horizontal axis through its center, as shown. The linear speed of a passenger on the rim is constant and equal to 6 m/s. What are the magnitude and direction of the passenger's acceleration as she passes through:

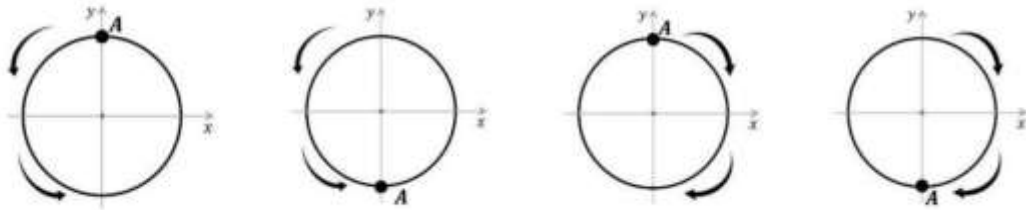


- (a) The lowest point in her circular motion.
- (b) The highest point in her circular motion.
- (c) How much time does it take the Ferris wheel to make one revolution?

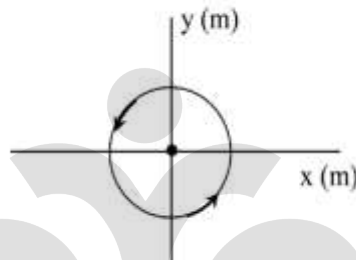
Q7: The shown fan rotates uniformly and takes 0.5 s to make one revolution (blade length = 15 cm). Find the fan's radial acceleration (in m/s^2).



Q8: A particle is moving in a horizontal circular path at a constant speed. If the velocity and acceleration of the particle at point A are $\vec{v}_A = -4\hat{i}$ m/s and $\vec{a}_A = -2\hat{j}$ m/s², respectively, identify the figure (among the four choices shown) that satisfies these conditions.



Q9: A particle moves counterclockwise in a horizontal circle of radius 2 m, centered on the origin, as shown. At a certain instant the velocity and acceleration of the particle are $\vec{V} = -6\hat{j}$ m/s and $\vec{a} = (3\hat{i} + 4\hat{j})$ m/s², respectively.



- (a) What is the position vector $\vec{r}$ of the particle at this instant, in unit vector notation?
 (b) The speed of the particle at this instant is: increasing, decreasing, or constant?

Question 1

$\mathbf{r}(t) = [(6+3t)\hat{i} + (5+2t-2t^2)\hat{j}]$ m. Find (a) $\mathbf{r}$ and $\mathbf{v}$ at $t=0$ (b) $\mathbf{a}$ (c) time of maximum y .

$$\mathbf{v}(t) = d\mathbf{r}/dt = 3\hat{i} + (2-4t)\hat{j} \text{ (m/s)}$$

(a) $\mathbf{r}(0) = 6\hat{i} + 5\hat{j}$ m $\mathbf{v}(0) = 3\hat{i} + 2\hat{j}$ m/s

(b) $\mathbf{a}(t) = d\mathbf{v}/dt = -4\hat{j}$ m/s² (constant)

(c) max $y \rightarrow v_y = 0: 2 - 4t = 0 \rightarrow t = 2/4$

✓ **Answer:** $\mathbf{r}(0)=6\mathbf{i}+5\mathbf{j}$ m, $\mathbf{v}(0)=3\mathbf{i}+2\mathbf{j}$ m/s, $\mathbf{a}=-4\mathbf{j}$ m/s², $t=0.5$ s

Question 2

From the origin: $\mathbf{v}_0=(3\hat{i} + 12\hat{j})$ m/s, $\mathbf{a}=(1\hat{i} - 2\hat{j})$ m/s² (constant).

Find (a) position at $t=3$ s (b) $\mathbf{v}$ and speed at $t=2$ s (c) time $v_y=0$.

(a) $\mathbf{r}(t) = \mathbf{v}_0 t + (1/2)\mathbf{a} t^2$

$$\mathbf{r}(3) = [3(3)+(1/2)(1)(3)^2]\hat{i} + [12(3)+(1/2)(-2)(3)^2]\hat{j} = [9+4.5]\hat{i} + [36-9]\hat{j}$$

(b) $\mathbf{v}(t) = \mathbf{v}_0 + \mathbf{a} t \rightarrow \mathbf{v}(2) = [3+1(2)]\hat{i} + [12-2(2)]\hat{j} = 5\hat{i} + 8\hat{j}$ m/s

$$\text{speed} = \sqrt{(5^2 + 8^2)} = \sqrt{(89)} = 9.43 \text{ m/s}$$

(c) $v_y = 0: 12 - 2t = 0 \rightarrow t = 12/2$

✓ **Answer:** (a) $\mathbf{r}(3)=13.5\mathbf{i}+27\mathbf{j}$ m (b) $\mathbf{v}(2)=5\mathbf{i}+8\mathbf{j}$ m/s, speed=9.43 m/s (c) $t=6.0$ s

Question 3

Ball leaves foot at A, 0.95 m above ground, $v_0=15$ m/s at 36.9° above horizontal. Find (a) time to reach the ground (b) horizontal distance d (c) velocity at B.

$$v_{0x} = 15 \cos 36.9^\circ = 15(0.8) = 12 \text{ m/s}$$

$$v_{0y} = 15 \sin 36.9^\circ = 15(0.6) = 9 \text{ m/s}$$

(a) $y(t) = 0.95 + 9t - 5t^2 = 0$

$$\rightarrow 5t^2 - 9t - 0.95 = 0$$

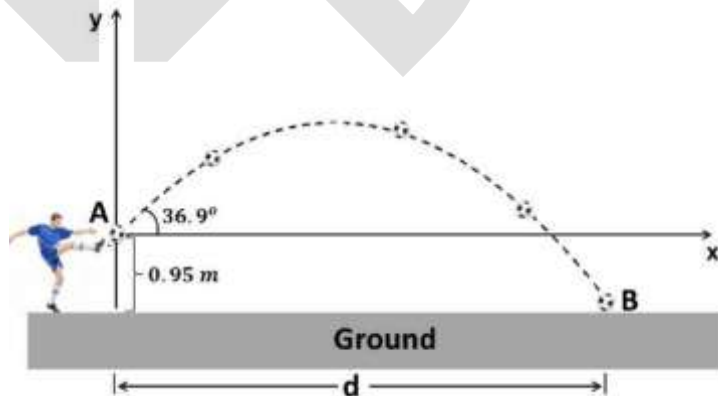
$$t = [9 + \sqrt{(9^2 + 4(5)(0.95))}] / (2 \times 5)$$

$$= [9 + \sqrt{(100)}] / 10 = [9 + 10] / 10 = 1.9 \text{ s}$$

(b) $d = v_{0x} \times t = 12 \times 1.9 = 22.8 \text{ m}$

(c) $v_x = v_{0x} = 12 \text{ m/s}$ (unchanged)

$$v_y = v_{0y} - g t = 9 - 10(1.9) = 9 - 19 = -10 \text{ m/s}$$



✓ **Answer:** (a) $t=1.9$ s (b) $d=22.8$ m (c) $\mathbf{v}_B = 12\mathbf{i} - 10\mathbf{j}$ m/s

Question 4

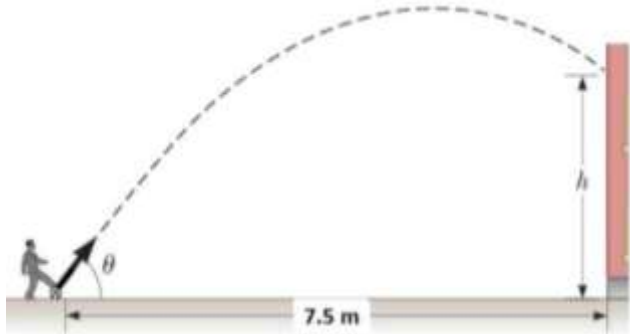
Ball kicked at angle θ , $v_0=14$ m/s, reaches a wall 7.5 m away in $t=2$ s. Find the height h at the wall.

horizontal (no acceleration):

$$7.5 = v_{0x} \times t = v_{0x}(2) \rightarrow v_{0x} = 7.5/2$$

$$v_{0y} = \sqrt{(v_0^2 - v_{0x}^2)} = \sqrt{(14^2 - 3.75^2)} \\ = 13.49 \text{ m/s}$$

$$h = v_{0y} t - (1/2)g t^2 = 13.49(2) - (1/2)(10)(2)^2 \\ = 26.98 - 20 = 6.98 \text{ m}$$



✓ Answer: $h = 6.98 \text{ m}$ (approx. 7.0 m)

Question 5

Cannon fires 36.9° below horizontal from a cliff. Ball hits the ground after $t=2$ s at horizontal distance 48m. Find (a) initial speed (b) velocity at impact (c) cliff height.

(a) horizontal:

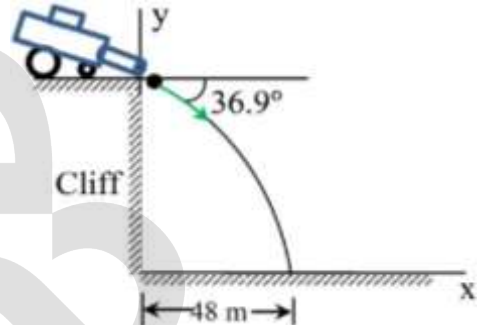
$$48 = v_0 \cos 36.9^\circ \times t = v_0(0.8)(2) = 1.6 v_0 \\ \rightarrow v_0 = 48/1.6 = 30 \text{ m/s}$$

$$v_{0x} = 30(0.8) = 24 \text{ m/s}$$

$$v_{0y} = -30 \sin 36.9^\circ = -30(0.6) = -18 \text{ m/s (below horizontal)}$$

(b) $v_x = v_{0x} = 24 \text{ m/s}$ $v_y = v_{0y} - g t = -18 - 10(2) = -38 \text{ m/s}$

(c) height fallen: $\Delta Y = v_{0y} t - (1/2)g t^2 = -18(2) - (1/2)(10)(2)^2 = -36 - 20 = -56 \text{ m}$



✓ Answer: (a) $v_0=30 \text{ m/s}$ (b) $v=24i-38j \text{ m/s}$ (c) cliff height = 56 m

Question 6

Ferris wheel, radius=14 m, constant speed $v=6$ m/s. Find the acceleration at

(a) lowest point (b) highest point (c) period of one revolution.

$$a = v^2/R = 6^2/14 = 36/14 = 2.57 \text{ m/s}^2$$

(magnitude is the same at every point - only the direction changes, always toward the center)

(c) $T = 2\pi R / v = 2\pi(14)/6 = 14.7 \text{ s}$



✓ Answer: (a) 2.57 m/s^2 , directed upward (toward the center)
(b) 2.57 m/s^2 , directed downward (toward the center) (c) $T=14.7 \text{ s}$

Question 7

Fan rotates uniformly, period=0.5 s, blade length $r=15\text{ cm}=0.15\text{ m}$.
Find the radial acceleration.



$$\omega = 2\pi / T = 2\pi / 0.5 = 4\pi \text{ rad/s}$$

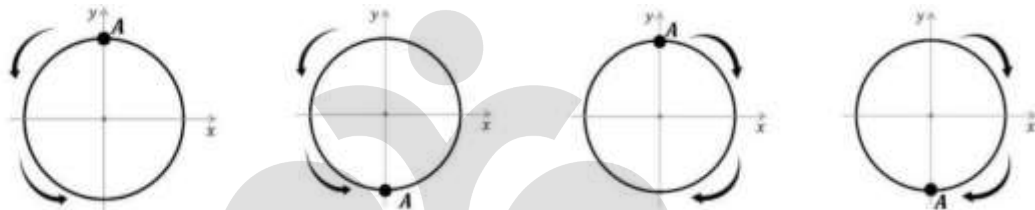
$$a = \omega^2 r = (4\pi)^2 (0.15) = (12.57)^2 (0.15)$$

✓ **Answer: $a = 23.7\text{ m/s}^2$**

Question 8

Particle moves on a horizontal circle at constant speed. At point A: $v_A = -4\hat{i}\text{ m/s}$, $a_A = -2\hat{j}\text{ m/s}^2$.

Which of the four diagrams matches?



constant speed $\rightarrow$ acceleration is purely centripetal, i.e. a always points from A toward the center

$a_A = -2\hat{j}$ (points in $-y$) $\rightarrow$ center is below A $\rightarrow$ A is at the TOP of the circle

at the top, $v_A = -4\hat{i}$ (points in $-x$) $\rightarrow$ matches counterclockwise rotation (for CCW motion, velocity at the top points in $-x$)

✓ **Answer: The 1st diagram (A at top, rotating counterclockwise) is correct.**

Question 9

Particle moves counterclockwise on a circle of radius 2 m centered at the origin.

At an instant: $v = -6\hat{j}\text{ m/s}$, $a = (3\hat{i} + 4\hat{j})\text{ m/s}^2$.

Find (a) position vector r ?

(b) is the speed increasing, decreasing, or constant?

solution

(a) v is always tangent to the circle, so

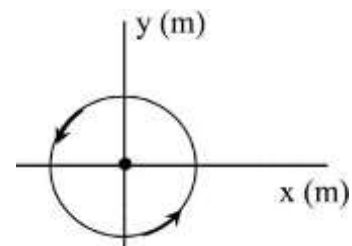
$$r \cdot v = 0 \text{ With } v = (0, -6): r \cdot v = -6r_y = 0 \rightarrow r_y = 0 \rightarrow r = (\pm 2, 0)$$

counterclockwise $\rightarrow (r \times v)_z > 0$: for $r = (-2, 0)$: $r_x v_y - r_y v_x$

$$= (-2)(-6) - 0 = 12 > 0 \text{ (correct sign)}$$

(b) $a \cdot v = (3)(0) + (4)(-6) = -24 < 0 \rightarrow$ acceleration has a component opposite to v

✓ **Answer: (a) $r = (-2\hat{i} + 0\hat{j})\text{ m}$ (b) speed is decreasing**



--- End of solutions ---

Book Questions Chapter 3

Exercises

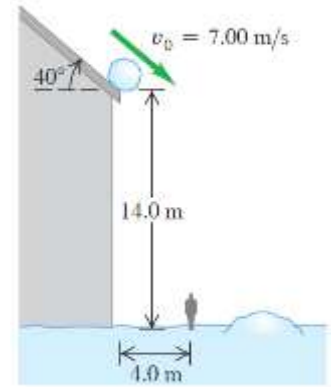
- 3.3:** CALC A web page designer creates an animation in which a dot on a computer screen has position $\mathbf{r} = [4.4 \text{ cm} + (2.8 \text{ cm/s}^2)t^2] \hat{i} + (5.5 \text{ cm/s})t \hat{j}$.
- (a) Find the magnitude and direction of the dot's average velocity between $t = 0$ and $t = 2.0 \text{ s}$.
 - (b) Find the magnitude and direction of the instantaneous velocity at $t = 0$, $t = 1.0 \text{ s}$, and $t = 2.0 \text{ s}$.
 - (c) Sketch the dot's trajectory from $t = 0$ to $t = 2.0 \text{ s}$, and show the velocities calculated in part (b).
- 3.5:** A jet plane is flying at a constant altitude. At time $t_1 = 0$, it has components of velocity $v_x = 88 \text{ m/s}$, $v_y = 115 \text{ m/s}$. At time $t_2 = 30.0 \text{ s}$, the components are $v_x = -175 \text{ m/s}$, $v_y = 35 \text{ m/s}$.
- (a) Sketch the velocity vectors at t_1 and t_2 . How do these two vectors differ?
 - (b) For this time interval, calculate the components of the average acceleration.
 - (c) Find the magnitude and direction of the average acceleration.
- 3.9:** A physics book slides off a horizontal tabletop with a speed of 1.40 m/s . It strikes the floor in 0.320 s . Ignore air resistance. Find:
- (a) The height of the tabletop above the floor.
 - (b) The horizontal distance from the edge of the table to the point where the book strikes the floor.
 - (c) The horizontal and vertical components of the book's velocity, and the magnitude and direction of its velocity, just before the book reaches the floor.
 - (d) Draw x - t , y - t , v_x - t , and v_y - t graphs for the motion.
- 3.11:** Crickets Chirpy and Milada jump from the top of a vertical cliff. Chirpy drops downward and reaches the ground in 3.10 s , while Milada jumps horizontally with an initial speed of 94.0 cm/s . How far from the base of the cliff will Milada hit the ground? Ignore air resistance.
- 3.16:** On level ground a shell is fired with an initial velocity of 71.0 m/s at 55.9° above the horizontal and feels no appreciable air resistance.
- (a) Find the horizontal and vertical components of the shell's initial velocity.
 - (b) How long does it take the shell to reach its highest point?
 - (c) Find its maximum height above the ground.
 - (d) How far from its firing point does the shell land?
 - (e) At its highest point, find the horizontal and vertical components of its acceleration and velocity.
- 3.21:** A man stands on the roof of a 15.0-m -tall building and throws a rock with a speed of 30.0 m/s at an angle of 33.0° above the horizontal. Ignore air resistance. Calculate:
- (a) The maximum height above the roof that the rock reaches.
 - (b) The speed of the rock just before it strikes the ground.
 - (c) The horizontal range from the base of the building to the point where the rock strikes the ground.
 - (d) Draw x - t , y - t , v_x - t , and v_y - t graphs for the motion.

3.34: The radius of the earth's orbit around the sun (assumed to be circular) is 1.50×10^8 km, and the earth travels around this orbit in 365 days.

- (a) What is the magnitude of the orbital velocity of the earth, in m/s?
- (b) What is the magnitude of the radial acceleration of the earth toward the sun, in m/s^2 ?
- (c) Repeat parts (a) and (b) for the motion of the planet Mercury (orbit radius= 5.79×10^7 km, orbital period=88 days).

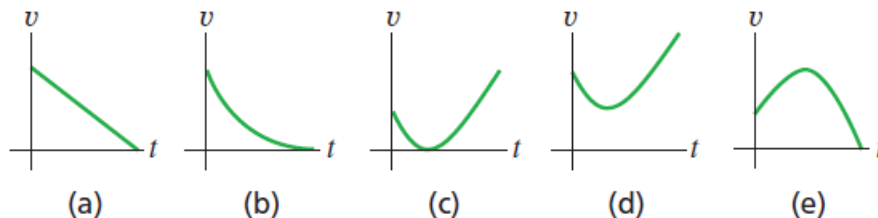
3.61: Look Out! A snowball rolls off a barn roof that slopes downward at an angle of 40° (Fig. P3.61). The edge of the roof is 14.0 m above the ground, and the snowball has a speed of 7.00 m/s as it rolls off the roof. Ignore air resistance.

- (a) How far from the edge of the barn does the snowball strike the ground if it doesn't strike anything else while falling?
- (b) Draw x - t , y - t , v_x - t , and v_y - t graphs for the motion in part (a).
- (c) A man 1.9 m tall is standing 4.0 m from the edge of the barn. Will the snowball hit him?



Questions

- Q3.1:** A simple pendulum (a mass swinging at the end of a string) swings back and forth in a circular arc. What is the direction of the acceleration of the mass when it is at the ends of the swing? At the midpoint? In each case, explain how you obtained your answer.
- Q3.6:** A package falls out of an airplane that is flying in a straight line at a constant altitude and speed. If you ignore air resistance, what would be the path of the package as observed by the pilot? As observed by a person on the ground?
- Q3.9:** A projectile is fired upward at an angle θ above the horizontal with an initial speed v_0 . At its maximum height, what are its velocity vector, its speed, and its acceleration vector?
- Q3.16:** A stone is thrown into the air at an angle above the horizontal and feels negligible air resistance. Which graph in Fig. Q3.16 best depicts the stone's speed v as a function of time t while it is in the air?



Exercises

3.3

$\mathbf{r} = [4.4 + 2.8t^2] \hat{i} + [5.5t] \hat{j}$ (cm, t in s). Find

(a) average velocity 0-2.0 s

(b) instantaneous velocity at t=0, 1.0, 2.0 s.

$$\mathbf{r}(0) = 4.4 \hat{i} \text{ cm} \quad \mathbf{r}(2.0) = [4.4 + 2.8(4.0)] \hat{i} + [5.5(2.0)] \hat{j} = 15.6 \hat{i} + 11.0 \hat{j} \text{ cm}$$

$$(a) \mathbf{v}_{\text{avg}} = \Delta \mathbf{r} / \Delta t = (15.6 - 4.4) \hat{i} / 2.0 + 11.0 \hat{j} / 2.0 = 5.6 \hat{i} + 5.5 \hat{j} \text{ cm/s}$$

$$|\mathbf{v}_{\text{avg}}| = \sqrt{(5.6^2 + 5.5^2)} = 7.85 \text{ cm/s} \quad \theta = \tan^{-1}(5.5/5.6) = 44.5^\circ$$

$$(b) \mathbf{v}(t) = d\mathbf{r}/dt = 5.6t \hat{i} + 5.5 \hat{j} \text{ cm/s}$$

$$\mathbf{v}(0) = 5.5 \hat{j} \text{ cm/s} \rightarrow 5.5 \text{ cm/s at } 90^\circ$$

$$\mathbf{v}(1.0) = 5.6 \hat{i} + 5.5 \hat{j} \text{ cm/s} \rightarrow 7.85 \text{ cm/s at } 44.5^\circ$$

$$\mathbf{v}(2.0) = 11.2 \hat{i} + 5.5 \hat{j} \text{ cm/s} \rightarrow 12.5 \text{ cm/s at } 26.2^\circ$$

(c) the path is a rightward-curving arc (x grows as t^2 , y grows linearly); the three velocity vectors above are tangent to it at t=0, 1.0, 2.0 s

✓ **Answer:** $\mathbf{v}_{\text{avg}} = 7.85 \text{ cm/s at } 44.5^\circ$. $\mathbf{v}(0) = 5.5 \text{ cm/s at } 90^\circ$, $\mathbf{v}(1.0) = 7.85 \text{ cm/s at } 44.5^\circ$, $\mathbf{v}(2.0) = 12.5 \text{ cm/s at } 26.2^\circ$

3.5

$t_1=0$: $v_x=88 \text{ m/s}$, $v_y=115 \text{ m/s}$. $t_2=30.0 \text{ s}$: $v_x=-175 \text{ m/s}$, $v_y=35 \text{ m/s}$. Find the average acceleration.

(a) the two velocity vectors differ in both magnitude and direction - the plane has turned and changed speed

$$(b) \mathbf{a}_{x\text{-avg}} = (v_{x2} - v_{x1}) / (t_2 - t_1) = (-175 - 88) / 30.0 = -8.77 \text{ m/s}^2$$

$$\mathbf{a}_{y\text{-avg}} = (v_{y2} - v_{y1}) / (t_2 - t_1) = (35 - 115) / 30.0 = -2.67 \text{ m/s}^2$$

$$(c) |\mathbf{a}_{\text{avg}}| = \sqrt{(8.77^2 + 2.67^2)} \quad \theta = 180^\circ + \tan^{-1}(2.67/8.77) = 16.93^\circ$$

(both components negative, so the vector lies in the 3rd quadrant add 180 to counted from +X CCW)

✓ **Answer:** $\mathbf{a}_{\text{avg}} = -8.77\hat{i} - 2.67\hat{j} \text{ m/s}^2$, $|\mathbf{a}_{\text{avg}}| = 9.17 \text{ m/s}^2 \text{ at } 196.9^\circ \text{ from +x-axis \& CCW}$

3.9

Book slides off a table at 1.40 m/s (horizontal) and strikes the floor after 0.320 s.

Find (a) table height (b) horizontal distance (c) velocity components and speed at impact.

$$(a) h = (1/2)gt^2 = (1/2)(10)(0.320)^2 = 0.512 \text{ m}$$

$$(b) \Delta x = v_0 t = 1.40(0.320) = 0.448 \text{ m}$$

$$(c) v_x = v_0 = 1.40 \text{ m/s (unchanged)} \quad v_y = gt = 10(0.320) = 3.20 \text{ m/s (downward)}$$

$$\text{speed} = \sqrt{(1.40^2 + 3.20^2)} = 3.49 \text{ m/s} \quad \theta = \tan^{-1}(3.20/1.40) = 66.4^\circ \text{ below the horizontal}$$

$$(d) x(t) = 1.40t, \quad y(t) = -5t^2, \quad v_x(t) = 1.40 \text{ (constant)}, \quad v_y(t) = -10t$$

✓ **Answer:** (a) $h=0.512 \text{ m}$ (b) $\Delta x=0.448 \text{ m}$
(c) $v_x=1.40 \text{ m/s}$, $v_y=3.20 \text{ m/s downward}$,
speed= $3.49 \text{ m/s at } 66.4^\circ \text{ below horizontal}$

3.11

Chirpy drops straight down and lands in 3.10 s. Milada jumps horizontally at 94.0 cm/s from the same cliff. How far from the base does Milada land?

same height → same fall time (horizontal launch does not change vertical motion): $t = 3.10$ s

$$x = v_0 \times t = 0.940 \times 3.10$$

✓ **Answer: $x = 2.91$ m**

3.16

Shell fired at 71.0 m/s, 55.9° above horizontal, no air resistance. Find (a) v_{0x} , v_{0y} (b) time to highest point (c) max height (d) range (e) v and a at highest point.

(a) $v_{0x} = 71.0 \cos 55.9^\circ = 39.8$ m/s $v_{0y} = 71.0 \sin 55.9^\circ = 58.8$ m/s

(b) $t_{\text{top}} = v_{0y} / g = 58.8 / 10 = 5.88$ s

(c) $h_{\text{max}} = v_{0y}^2 / (2g) = 58.8^2 / 20 = 172.9$ m

(d) $R = 2 v_{0x} v_{0y} / g = 2(39.8)(58.8) / 10 = 468$ m

(e) at the top: $v_x = v_{0x} = 39.8$ m/s, $v_y = 0$ $a_x = 0$, $a_y = -g = -10$ m/s²

✓ **Answer: $v_{0x}=39.8$, $v_{0y}=58.8$ m/s; $t=5.88$ s; $h_{\text{max}}=172.9$ m; $R=468$ m; at top: $v=39.8$ m/s (horizontal), $a=10$ m/s² downward**

3.21

Man on a 15.0-m building throws a rock at 30.0 m/s, 33.0° above horizontal.

Find (a) max height above the roof

(b) speed just before the ground

(c) horizontal range from the building.

$$v_{0x} = 30.0 \cos 33.0^\circ = 25.16 \text{ m/s} \quad v_{0y} = 30.0 \sin 33.0^\circ = 16.34 \text{ m/s}$$

(a) $h_{\text{max}} = v_{0y}^2 / (2g) = 16.34^2 / 20 = 13.3$ m

(b) $v_y^2 = v_{0y}^2 - 2g(\Delta y) = 16.34^2 - 2(10)(-15.0) = 566.99$

$$v_y = -\sqrt{(566.99)} = -23.81 \text{ m/s} \quad \text{speed} = \sqrt{(25.16^2 + 23.81^2)} = 34.6 \text{ m/s}$$

(c) $-15.0 = 16.34t - 5t^2 \rightarrow 5t^2 - 16.34t - 15.0 = 0 \rightarrow t = 4.015$ s

$$\text{range} = v_{0x} \times t = 25.16 \times 4.015 = 101.0 \text{ m}$$

(d) $x(t) = 25.16t$, $y(t) = 16.34t - 5t^2$, $v_x(t) = 25.16$ (constant), $v_y(t) = 16.34 - 10t$

✓ **Answer: (a) $h_{\text{max}}=13.3$ m (b) speed=34.6 m/s (c) range=101.0 m**

3.34

Earth's orbit: $r = 1.50 \times 10^8$ km, $T = 365$ days.

Find (a) orbital speed (b) radial acceleration. (c) Repeat for Mercury: $r = 5.79 \times 10^7$ km, $T = 88$ days.

Earth: $r = 1.50 \times 10^{11}$ m, $T = 365(86400) = 3.154 \times 10^7$ s

$$v = 2\pi r / T = 2\pi(1.50 \times 10^{11}) / (3.154 \times 10^7) = 2.99 \times 10^4 \text{ m/s}$$

$$a = v^2 / r = (2.989 \times 10^4)^2 / (1.50 \times 10^{11})$$

Mercury: $r = 5.79 \times 10^{10}$ m, $T = 88(86400) = 7.603 \times 10^6$ s

$$v = 2\pi(5.79 \times 10^{10}) / (7.603 \times 10^6) = 4.78 \times 10^4 \text{ m/s}$$

$$a = v^2 / r = (4.785 \times 10^4)^2 / (5.79 \times 10^{10}) = 3.95 \times 10^{-2} \text{ m/s}^2$$

✓ **Answer:** Earth: $v = 2.99 \times 10^4$ m/s, $a = 5.95 \times 10^{-3}$ m/s².

Mercury: $v = 4.78 \times 10^4$ m/s, $a = 3.95 \times 10^{-2}$ m/s²

3.61

Snowball rolls off a barn roof sloped 40° , edge 14.0 m above the ground, speed 7.00 m/s along the roof. (a) How far from the edge does it land? (c) Does it hit a 1.9-m-tall man standing 4.0 m from the edge?

$$v_{0x} = 7.00 \cos 40^\circ = 5.362 \text{ m/s} \quad v_{0y} = -7.00 \sin 40^\circ = -4.499 \text{ m/s}$$

$$(a) \ y(t) = v_{0y} t - (1/2)g t^2 = -14.0 \rightarrow 5t^2 + 4.499t - 14.0 = 0$$

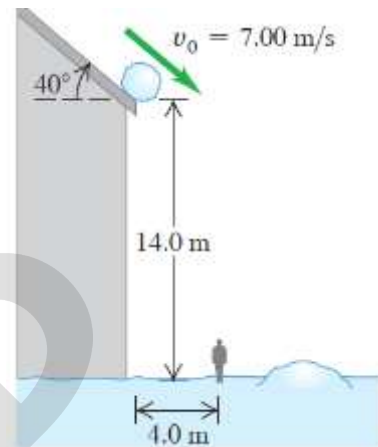
$$t = 1.283 \text{ s} \quad \text{Then, } x = v_{0x} \times t = 5.362 \times 1.283 = 6.88 \text{ m}$$

$$(b) \ x(t) = 5.362t, \ y(t) = -4.499t - 5t^2, \ v_x(t) = 5.362 \text{ (constant)}, \ v_y(t) = -4.499 - 10t$$

$$(c) \ \text{at } x = 4.0 \text{ m: } t = 4.0 / 5.362 = 0.746 \text{ s}$$

$$y(0.746) = -4.499(0.746) - 5(0.746)^2 = -3.36 - 2.78 = -6.14 \text{ m} \rightarrow$$

$$\text{height above ground} = 14.0 - 6.14$$



✓ **Answer:** (a) lands 6.88 m from the edge.

(c) height over the man = 7.86 m >> 1.9 m - it passes well above him, so it does NOT hit him.

Questions

Q3.1

A pendulum swings back and forth. What is the direction of its acceleration at the ends of the swing? At the midpoint?

at the ends: $v=0 \rightarrow$ centripetal part (v^2/r) = 0 $\rightarrow$ only the tangential part remains, directed tangent to the arc, toward the midpoint

at the midpoint: speed is maximum (momentarily not changing) $\rightarrow$ tangential part = 0 $\rightarrow$ only the centripetal part remains, directed straight up toward the pivot

✓ **Answer:** Ends: tangent to the arc, pointing toward the midpoint.

Midpoint: purely centripetal, pointing straight up toward the pivot.

Q3.6

A package falls from a plane flying level at constant speed (ignore air resistance). What path does it trace, as seen by (a) the pilot, (b) a person on the ground?

package keeps the plane's horizontal velocity $\rightarrow$ relative to the plane, its horizontal velocity is always zero; only a vertical (downward) motion remains

relative to the ground: constant horizontal velocity + vertical acceleration $g \rightarrow$ parabolic path

✓ **Answer:** Pilot: a straight vertical line, directly below the plane. Ground observer: a parabola.

Q3.9

A projectile is fired at angle θ with initial speed v_0 . At its maximum height, what are its velocity, speed, and acceleration?

at the top: $v_y = 0 \rightarrow v = v_0 \cos\theta \hat{i}$ (purely horizontal)

speed = $v_0 \cos\theta$ (minimum value, but not zero)

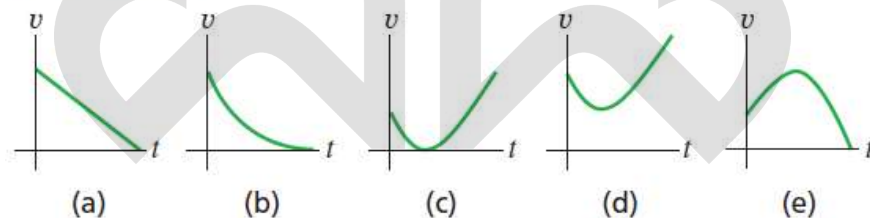
$a = -g \hat{j}$ (gravity never turns off, even at the top - the vertical velocity is zero, not the acceleration)

✓ **Answer:** $v = v_0 \cos(\theta)$, horizontal;
speed = $v_0 \cos(\theta)$; $a = g$, straight down (unchanged).

Q3.16

A stone is thrown at an angle above the horizontal. Which graph in Fig. Q3.16 best shows its speed v vs. time t ?

Figure Q3.16



$v_x = v_0 \cos\theta = \text{constant}$ (never zero)

$v_y(t) = v_0 \sin\theta - g t$ (decreases, crosses zero at the top, then grows negative)

$v(t) = \sqrt{v_x^2 + v_y(t)^2} \rightarrow v$ is largest at launch and landing, and dips to a MINIMUM (equal to v_x , not zero) at the top

✓ **Answer:** (c) - speed decreases to a nonzero minimum at the top, then increases again.

--- End of solutions ---